Chapter 3 Review Name Key

Pre-Calc per. 2015/2016

Find the quotient and remainder using synthetic division

1. $\frac{x^4 - 4x^2 + 2x + 5}{x - 2}$

$2 \mid 1 \quad 0 \quad -4 \quad 2 \quad 5$

$\quad\quad\; 2 \quad 4 \quad 0 \quad 4$

$\quad 1 \quad 2 \quad 0 \quad 2 \quad 9$

Q: $1x^3 + 2x^2 + 2$

R: 9

2. $\frac{x^3 + 3}{x + 3}$

$-3 \mid 1 \quad 0 \quad 0 \quad 3$

$\quad\quad\; -3 \quad 9 \quad -27$

$\quad 1 \quad -3 \quad 9 \quad -24$

Q: $x^2 - 3x + 9$

R: -24

Find the quotient and remainder using long division

3. $(-x^3 + 2x^2 - 2x + 3) \div (x^2 - 1)$

$$\begin{array}{r} -1x + 2 \\ x^2 + 0x - 1 \overline{)\, -1x^3 + 2x^2 - 2x + 3} \\ -(-1x^3 + 0x^2 + 1x) \\ \hline 2x^2 - 3x + 3 \\ -(2x^2 + 0x - 2) \\ \hline -3x + 5 \end{array}$$

Q: $-1x + 2$

R: $-3x + 5$

4. $\frac{2x^5 + 4x^4 - x^3 - x^2 + 7}{2x^2 - 1}$

$$\begin{array}{r} x^3 + 2x^2 + 1/2 \\ 2x^2 + 0x - 1 \overline{)\, 2x^5 + 4x^4 - x^3 - x^2 + 0x + 7} \\ 2x^5 + 0x^4 - x^3 \\ \hline 4x^4 - 0x^3 - x^2 + 0x + 7 \\ +(4x^4 + 0x^3 + 2x^2) \\ \hline 1x^2 + 0x + 7 \\ -(1x^2 + 0x - 1/2) \\ \hline 7\,1/2 \end{array}$$

Q: $x^3 + 2x^2 + 1/2$

R: $7\,1/2$

5. Show that ½ is a zero of the polynomial

$P(x) = 2x^4 + x^3 - 5x^2 + 10x - 4$

$P(1/2) = 2(1/2)^4 + (1/2)^3 - 5(1/2)^2 + 10(1/2) - 4$

$2 \cdot \frac{1}{16} + \frac{1}{8} - 5 \cdot \frac{1}{4} + 5 - 4$

$\frac{1}{8} + \frac{1}{8} - \frac{5}{4} + 5 - 4$

$\frac{2}{8} - \frac{10}{8} + \frac{40}{8} - \frac{32}{8} = 0$ ✓

6. Perform the indicated operation and write the result in the form a + bi

a) $(3 - 2i) + (4 + 3i)$

$7 + i$

b) $(3 - 2i) - (4 + 3i)$

$-1 - 5i$

c) $(3 - 2i)(4 + 3i)$

$12 + 9i - 8i - 6i^2$

$12 + i + 6$

$18 + i$

d) $\frac{3-2i}{4+3i}\cdot\frac{(4-3i)}{(4-3i)}$

$\frac{12 - 9i - 8i + 6i^2}{12 - 9i^2} = \frac{6 - 17i}{25}$

e) i^{48}

$(i^4)^{12} = (1)^{12} = 1$

f) $(\sqrt{2} - \sqrt{-2})(\sqrt{8} + \sqrt{-2})$

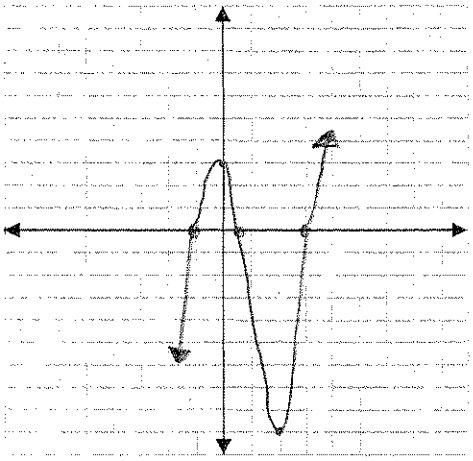
$(\sqrt{2} - i\sqrt{2})(\sqrt{8} + i\sqrt{2})$

$\sqrt{16} + 2i - i\sqrt{16} - 2i^2$

$4 + 2i - 4i + 2$

$6 - 2i$

7. Let $P(x) = 2x^3 - 5x^2 - 4x + 3$.



a. List all possible rational zeros of P.

$\pm 1, \pm 3, \pm 1/2, \pm 1/3$

b. Find the complete factorization of P.

$-1 \mid 2 \quad -5 \quad -4 \quad 3$

$\quad\quad -2 \quad 7 \quad -3$

$\quad 2 \quad -7 \quad 3 \quad 0$

$(x+1)(2x^2 - 7x + 3) = (x+1)(2x - 1)(x - 3)$

c. Find all zeros of P.

$3, -1, 1/2$

d. Sketch the graph of P.

x	y
-2	25
0	3
2	-9
4	35

8. Find the third-degree polynomial with integer coefficients that has zeros 2, and 3 – i, through the point (0, -40).

$(x - 2)(x - (3 - i))(x - (3 + i))$

$(x-2)(x-3+i)(x-3-i)$

$(x-2)(x^2 - 3x - xi - 3x + 9 + 3i + xi - 3i - i^2)$

$(x-2)(x^2 - 6x + 10)$

$x^3 - 6x^2 + 10x - 2x^2 + 12x - 20$

$c(y^3 - 8y^2 + 22y - 20)$

$P(x) = 2x^3 - 16x^2 + 44x - 40$

9. Let $P(x) = = x^4 + 3x^3 - 13x^2 - 15x$

$x(x^3 + 3x^2 - 13x - 15)$

a. List all possible rational zeros of P.

$\pm 1, \pm 3, \pm 5, \pm 15$

b. Find the complete factorization of P.

$-1 \mid 1 \quad 3 \quad -13 \quad -15$

$\quad\quad -1 \quad -2 \quad 15$

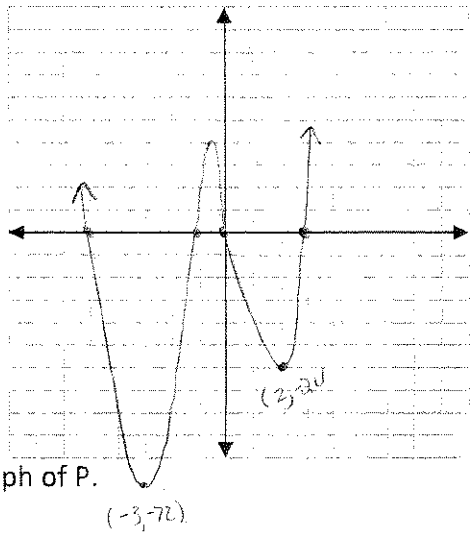
$\quad 1 \quad 2 \quad -15 \quad | \quad 0$

$(x)(x^2 + 2x - 15)$

$(x)(x+5)(x-3)$

c. Find all zeros of P.

$0, -5, 3, -1$



d. Sketch the graph of P.

x	y
-3	-72
$-\frac{1}{2}$	3.9
2	-42

10. Find all real and complex zeros of $P(x) = x^3 - x^2 - 4x - 6$.

$\pm 1, \pm 2, \pm 3, \pm 6$

$3 \mid 1 \quad -1 \quad -4 \quad -6$

$\quad\quad 3 \quad 6 \quad 6$

$\quad 1 \quad 2 \quad 2 \quad | \quad 0$

$\frac{-2 \pm \sqrt{4 - 4 \cdot 1 \cdot 2}}{2}$

Zeros: $3, -1+i, -1-i$

11. Find all the real and complex zeros of $P(x) = x^4 - 2x^3 + 5x^2 - 8x + 4$

$\pm 1, \pm 2, \pm 4$

$1 \mid 1 \quad -2 \quad 5 \quad -8 \quad 4$

$\quad\quad 1 \quad -1 \quad 4 \quad -4$

$1 \mid 1 \quad -1 \quad 4 \quad -4 \quad | \quad 0$

$\quad\quad 1 \quad 0 \quad 4$

$\quad 1 \quad 0 \quad 4 \quad | \quad 0$

$x^2 + 4 = 0$

$x^2 = -4$

$x = \pm 2i$

Zeros: 1 (with a mult. of 2), $\pm 2i$

12. Find a fourth-degree polynomial with integer coefficients that has zeros 3i and -1, with -1 a zero multiplicity of 2 and a leading coefficient of 5.

$(x-3i)(x+3i)(x+1)(x+1)$

$(x^2 - 9i^2)(x^2 + 2x + 1)$

$(x^2 + 9)(x^2 + 2x + 1)$

$x^4 + 2x^3 + x^2 + 9x^2 + 18x + 9$

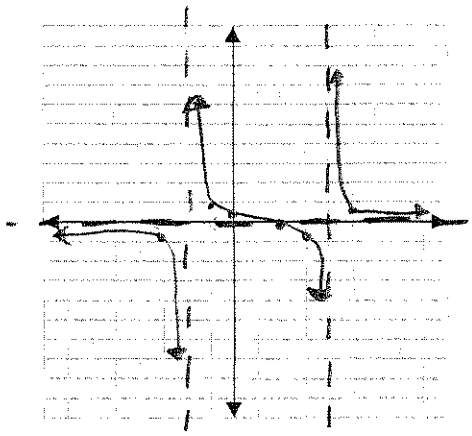
$5(x^4 + 2x^3 + 10x^2 + 18x + 9)$

$5x^4 + 10x^3 + 50x^2 + 90x + 45$

13. Graph the rational function. Show clearly all x- and y-intercepts and asymptotes.

a. $r(x) = \dfrac{x-2}{x^2-2x-8} = \dfrac{x-2}{(x-4)(x+2)}$

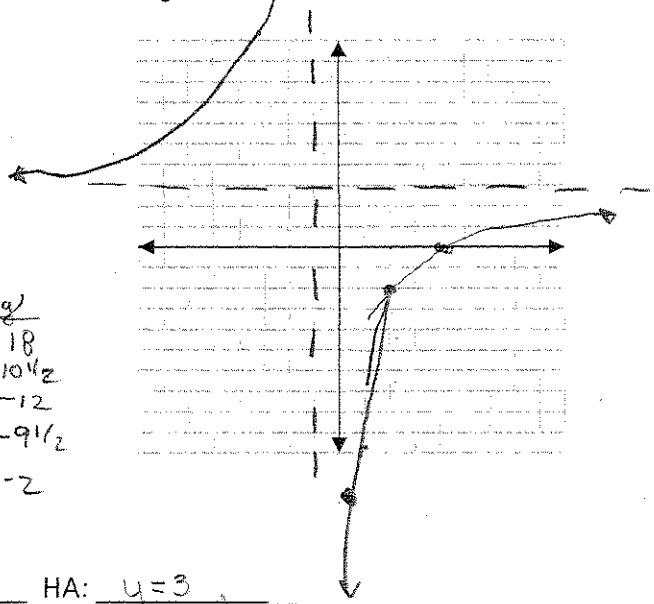
x	y
-3	5/10
-1	3/5
3	-1/5
6	1/25



x-intercept(s): (2, 0) y-intercept: (0, 1/4) VA: x=4; x=-2 HA: y = 0 bigger on bottom

b. $r(x) = \dfrac{3x-12}{x+1} = \dfrac{3(x-4)}{x+1}$

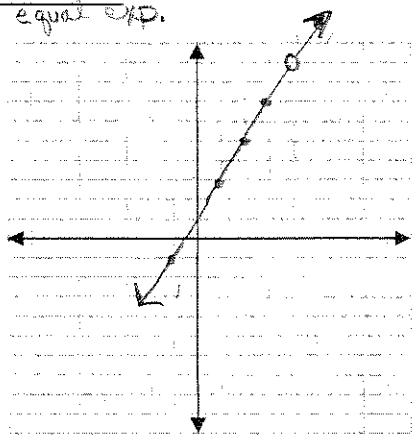
x	y
-2	18
-3	10½
0	-12
1	-9½
2	-2



x-intercept(s): (4, 0) y-intercept: (0, -12) VA: x=-1 HA: y=3 equal exp.

c. $r(x) = \dfrac{2x^2-7x-4}{x-4} = \dfrac{(2x+1)(x-4)}{x-4} = 2x+1 \quad x \neq 4$

x	y
0	1
1	3
-1	-1
-2	-3



x-intercept(s): (-½, 0) y-intercept: (0, 1) VA: — HA: — hole @ 4, 9