**Chapter 4 Review** Name Key

Pre-Calculus Per. ______

Let $f(x) = 3^x$ and evaluate the following:

1) $f(3)$ $3^3 = \boxed{27}$

2) $f\left(-\frac{1}{5}\right)$ $\boxed{.8027}$

3) $f(-\pi)$ $3^{-\pi} = \boxed{.0317}$

4) $f(\sqrt{2})$ $3^{\sqrt{2}} = \boxed{4.7289}$

Find the exponential function $f(x) = a^x$ that passes through the given point

5) (3, 125)
$125 = a^3$
$a = 5$
$\boxed{f(x) = 5^x}$

6) (-2, 16)
$16 = a^{-2}$
$a = \frac{1}{4}$
$\boxed{f(x) = \left(\frac{1}{4}\right)^x}$

Explain how the graph of $f(x) = 4^x$ would be moved

7) $g(x) = 3 + 4^x$
3 units up

8) $h(x) = -4^x + 2$
reflect about x-axis and 2 units up

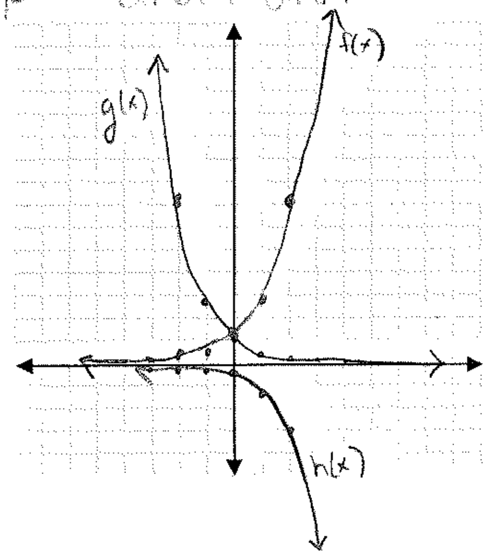
9) $k(x) = 4^{x-3} - 1$
3 units right and 1 unit down

Draw the graph of each function.

10) $g(x) = e^{-x}$

11) $h(x) = -\frac{1}{2}e^x$

| x | $f(x) = e^x$ | $g(x) = e^{-x}$ | $h(x) = -\frac{1}{2}e^x$ |
|---|---|---|---|
| -3 | .04979 | 20.086 | -.0249 |
| -2 | .13534 | 7.3891 | -.0677 |
| -1 | .36788 | 2.7183 | -.1839 |
| 0 | 1 | 1 | -.5 |
| 1 | 2.7183 | .36788 | -1.359 |
| 2 | 7.3891 | .13534 | -3.695 |
| 3 | 20.086 | .04979 | -10.04 |




Rewrite in Exponential Form

12) $\log_3 729 = 6$
$3^6 = 729$

13) $\log_4 25 = x$
$4^x = 25$

14) $\log 2x = y$
$10^y = 2x$

15) $\ln x = 20$
$e^{20} = x$

Rewrite in Logarithmic Form

16) $2^3 = 8$ $\log_2 8 = 3$

17) $\left(\frac{1}{3}\right)^{-2} = 9$ $\log_{1/3} 9 = -2$

18) $25^{-1/2} = 1/5$
$\log_{25}(1/5) = -1/2$

19) $e^{2x} = y$
$\ln(y) = 2x$

Evaluate without a calculator

20) $\log_8 1$
$8^x = 1$ $\boxed{x=0}$

21) $10^{\log 90}$ $3^{\log_3 5}$
$\boxed{90}$

22) $\log 0.000001$
$10^x = .000001$
$\boxed{x=-7}$

23) $\log_4 8$
$4^x = 8$
$2^{2x} = 2^3$
$\boxed{x = 3/2}$

24) $\log_3\left(\frac{1}{27}\right)$
$3^x = \frac{1}{27}$
$3^x = 3^{-3}$
$\boxed{x=-3}$

25) $e^{2\ln 7}$
$e^{\log_e 7^2}$
$7^2 = \boxed{49}$

26) $\log_2 \sqrt{16}$
$2^x = (16)^{1/2}$
$2^x = (2^4)^{1/2}$
$2^x = 2^2$ $\boxed{x=2}$

27) $\log_3 27^{23}$
$3^x = 27^{23}$
$3^x = (3^3)^{23}$
$3^x = 3^{69}$ $\boxed{x=69}$

Solve the equation. Find the exact solution if possible (no calculator). Otherwise approximate to two decimals.

38) $\log_2(1-x)=4$

$2^4 = 1-x$

$16 = 1-x$

$\boxed{x=-15}$

39) $5^{5-3x}=25$

$\log_5 25 = 5-3x$

$2 = 5-3x$

$-3 = -3x$

$\boxed{x=1}$

40) $e^{3x/4}=10$

$\log_e(10) = \frac{3x}{4}$

$4 \cdot \ln(10) = 3x$

$\boxed{x=3.07}$

41) $3^{1-x}=9^{2x+5}$

$\log_3 9^{2x+5} = 1-x$

$(2x+5)2 = 1-x$

$4x+10 = 1-x$

$5x = -9$

$\boxed{x=-9/5}$

42) log x + log (x + 1) = log 12

$\log_{10} x(x+1) = \cancel{\log_{10}} 12$

$x^2+x=12$

$\boxed{x=3, \cancel{-4}}$

$x^2+x-12=0$

$(x-3)(x+4)$

43) $\log_8(x+5)-\log_8(x-2)=1$

$\log_8 \frac{x+5}{x-2} = 1$

$8^1 = \frac{x+5}{x-2}$

$8(x-2) = x+5$

$8x-16 = x+5$

$7x = 21$

$\boxed{x=3}$

44) $x^2e^{2x}+2xe^{2x}=8e^{2x}$

$e^{2x}(x^2+2x-8)=0$

$e^{2x}(x-2)(x+4)=0$

$e^{2x}=0$ $\boxed{x=2}$ $\boxed{x=-4}$

$\cancel{\log_e 0 = 2x}$

45) $2^{3^x}=5$

$\log_2 5 = 3^x$

$2.3219 = 3^x$

$\log_3(2.3219) = x$

$\boxed{x=.77}$

46) $2^{x-1}=10$

$\log_2 10 = x-1$

$3.3219 = x-1$

$\boxed{x=4.32}$

47) 5ln(3 - x) = 4

$\ln(3-x) = 4/5$

$\log_e(3-x) = 4/5$

$\boxed{x=.77}$

$e^{4/5} = 3-x$

$-.77 = -x$

48) $10^{x+3}=6^{2x}$

$\log_{10} 6^{2x} = x+3$

$2x(\log_{10} 6) = x+3$

$2x(.7782) = x+3$

$1.5563x = \cancel{x}+3$

$-1x \quad -x$

$.5563x = 3$

$\boxed{x=5.39}$

49) $\log_2(x+2)+\log_2(x-1)=2$

$\log_2 (x+2)(x-1) = 2$

$2^2 = x^2+x-2$

$4 = x^2+x-2$

$x^2+x-6=0$

$(x-2)(x+3)=0$

$\boxed{x=2 \quad \cancel{x=-3}}$

50) The initial size of a culture of bacteria is 1000. After one hour the bacteria count is 8000. $n(t)=n_0e^{rt}$

a) Find a function that models the population after t hours.

$8000=1000e^{r(1)}$ $8=e^r$ $\log_e(8)=r$ $r=2.08$ $\boxed{n(t)=1000e^{2.08t}}$

b) Find the population after 1.5 hours.

$n(t)=1000e^{2.08(1.5)} = \boxed{22646.38 \text{ bacteria}}$

c) When will the population reach 15,000?

$15{,}000=1000e^{2.08t}$ $15=e^{2.08t}$ $\log_e 15=2.08t$ $\boxed{t=1.3 \text{ hours}}$

51) Suppose that $12,000 is invested in a savings account paying 5.6% interest per year.

a. Write the formula for the amount in the account after t years if interest is compounded monthly.

$A(t)=P$

$A(t)=12{,}000\left(1+\frac{0.056}{12}\right)^{12t}$ where $t$ is in years

b. Find the amount in the account after 3 years if interest is compounded daily.

$A(t)=12{,}000\left(1+\frac{0.056}{365}\right)^{365t}$. $A(3)=12{,}000\left(1+\frac{.056}{365}\right)^{365(3)}=\boxed{\$14{,}195.06}$

c. How long will it take for the amount in the account to grow to $20,000 if interest is compounded semiannually?

$A(t)=12{,}000\left(1+\frac{0.056}{2}\right)^{2t}=12{,}000(1.028)^{2t}$

$20{,}000=12{,}000(1.028)^{2t}$

$1.6667=1.028^{2t}$

$\log_{1.028}1.6667=2t$ $\boxed{t=9.25 \text{ yrs}}$

52) A car engine runs at a temperature of 190°F. When the engine is turned off, it cools according to Newton's Law of Cooling with constant K = 0.0341, where the time is measured in minutes. Find the time needed for the engine to cool to 90°F if the surrounding temperature is 60°F.

$T(t)=T_s+D_0e^{-kt}$

$K=.0341$

$T_s=60$

$D_0=190-60=130$

$90=60+130e^{-.0341(t)}$

$30=130e^{-.0341t}$

$3/13=e^{-.0341t}$

$\ln(3/13)=-.0341t$

$\boxed{t=43 \text{ minutes}}$

53) A sample of bismuth-210 decayed to 33% of its original mass after 8 days.

a. Find the half-life of this element.

$m(t)=m_0e^{-rt}$ $.33=1e^{-r(8)}$ $\ln(.33)=-8r$ $r=.13858$

$r=\frac{\ln 2}{h}$

$r=\frac{\ln 2}{h}=.13858$ $\ln 2=.13858h$ $\boxed{h=5 \text{ days}}$

b. Find the mass remaining after 12 days.

$m(12)=1e^{-(\ln 2/5)(12)}=.19=$ $\boxed{19\% \text{ of its original mass}}$

54) If one earthquake has magnitude 6.5 on the Richter Scale, what is the magnitude of another quake that is 35 times as intense?

$I_L=35\cdot I_s$

$M_L=\log\frac{I_L}{S}$

$M_L=\log I_L-\log S$

$M_s=\log\frac{I_s}{S}$

$M_s=\log I_s-\log S$ $M_L=\log 35\cdot I_s-\log S$

$M_L=\log 35+\log I_s-\log S$

$I_s=6.5$

Attached