STAFFORD ELEMENTARY:
A PARENT'S GUIDE TO MATHEMATICS

Dear Families,

Our goal at Stafford is for every child to leave us as a thoughtful learner, a positive member of the school community and confident about themselves and their abilities. We want each child to feel like they can be successful at any task or challenge placed before them. Math is no exception to this desired outcome! We know that for some adults math can be a subject area that makes us nervous and where we do not always have a great deal of confidence. This will, at times, translate into statements such as, “In school, I was not good at math.” Critical to our approach to math is creating the conditions for each child where their skills and understanding grow and they look at themselves as mathematicians who can tackle even the most complex of problems. The hope is that this positive disposition becomes a solid foundation that carries a child through their entire school career.

Our own attitudes about math matter to our children. They look to us for guidance, support and encouragement. Math does look different today in comparison to when we were all children. Our focus on building a conceptual basis through a variety of strategies is one example of the difference. Thus, we have created this math handbook in order to build a basis from which parents can feel confident in helping support their child’s mathematical growth. The handbook is meant as an overview of the thinking that informs our instructional approach to mathematics. We are proud of our students work in mathematics and the intent of this book is to further bridge the connections between home and school.

Everyone can be successful at math

Our overarching goal for students is that they develop mathematical proficiency. We want them to build a strong conceptual base of understanding in which their understanding and ability to integrate mathematical ideas to a wide range of problems becomes second nature. It is critical that students become accurate, flexible and efficient thinkers in math, as in all areas of the curriculum. It is important for us to create classrooms as mathematical communities where students use logic and mathematical evidence as justification of their thinking and solutions. This happens when students experience problems that require mathematical reasoning, conjecturing, and problem solving. It is essential that students see connections of math ideas and applications. It is through this process that students become confident and competent mathematicians.

We believe in the power of student discourse and the value this brings to student understanding. We want students to have the opportunity to explain their thinking and the strategies they used to solve a problem. When there is strong mathematical content in classroom conversations, we believe that this makes a difference in the depth of student understanding.

This handbook will go more in-depth into our thinking about mathematics. It is our hope that this background will help you support your child in their mathematical growth. Stafford teachers are so thoughtful in their work with students and as a result our students do very well. Our on-going learning and understanding of mathematics is important and it models for our students the power of lifelong learning. Stafford teachers approach math instruction through the lens of wanting each student to think of themselves as a mathematician. This handbook is meant as a tool to help us with this goal. We appreciate your support and your willingness to join us on this journey of mathematical understanding.

This handbook was put together through the efforts of the Site Council, Stafford teachers and through the thoughtful work by Chris Geigle, math/science teacher at Athey Creek Middle School. The idea for this handbook came as a result of the positive feedback from our writing handbook. We hope that you find this resource useful and that we can continue our home-school partnership of fostering thoughtful, creative and confident learners Kindergarten through 5th grade.

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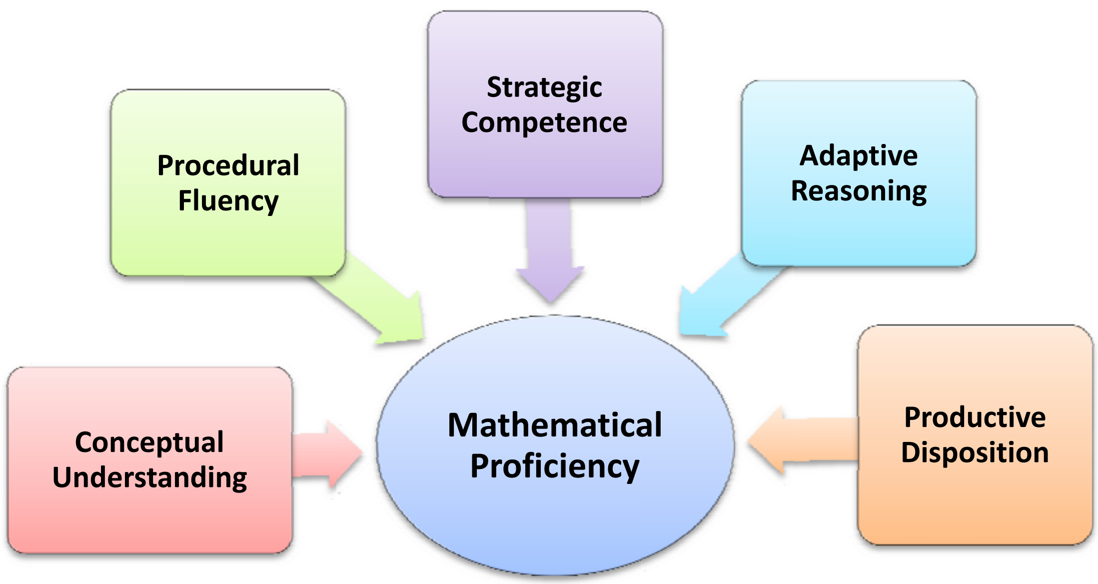
DEVELOPING MATHEMATICAL PROFICIENCY

Math in today's classrooms looks different than it has in the past. Schools have developed new methods to help children learn, understand and demonstrate their knowledge in all areas of the curriculum. This is especially true in mathematics.

Our mathematics instruction is based within framework of best practices. Best practices define a mathematical culture where students explain and justify their thinking through the use of inquiry, discussion, and collaboration. Students use models, manipulatives, and other mathematical tools to make sense of ideas, solve problems, and construct a solid foundation of mathematical understanding. By being given worthwhile tasks, students develop mathematical proficiency.

Being mathematically proficient means that a student displays certain behaviors and dispositions as they are "doing mathematics." There are five characteristics involved in being mathematically proficient: (1) conceptual understanding, (2) procedural fluency, (3) strategic competence, (4) adaptive reasoning, and (5) productive disposition. The last three of these characteristics only develop once students have experiences that involve these processes.



The five characteristics are interwoven and interdependent in the development of proficiency in mathematics. Mathematical proficiency is not a one-dimensional quality, and it cannot be achieved by focusing on just one or two of these strands. As students go from pre-kindergarten to fifth grade, they will become increasingly proficient in mathematics. That proficiency will enable them to successfully manage with the mathematical challenges of daily life and enable them to continue their study of mathematics in middle school and beyond.

In order to provide understanding of these five characteristics of mathematical proficiency, we will take a closer look at each component.

CONCEPTUAL UNDERSTANDING

Stafford students continually work toward a deeper, conceptual understanding of mathematics. By conceptual understanding, we mean students have an integrated and functional grasp of mathematical ideas; knowing more than just isolated facts and methods. They understand why a mathematical idea is important, the kinds of contexts in which it is useful and how to apply them effectively. For example, consider the task of multiplying 16 X 12. The graphics below are representations of a student's conceptual understanding of this problem.

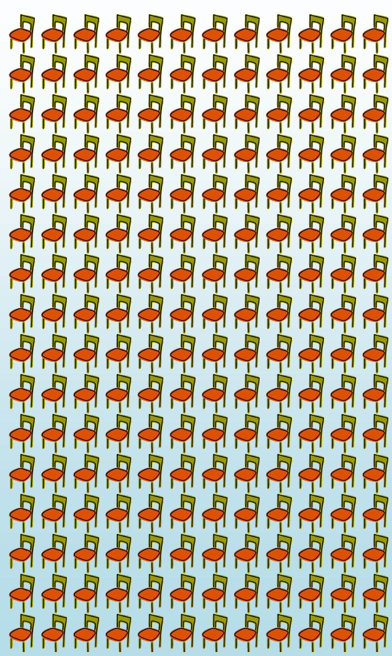
Facts and methods learned with understanding are connected, making them easier to remember and use.

Repeated addition: 12 + 12 + 12 + 12 + 12 + 12 + 12 + 12 + 12 + 12 + 12 + 12 + 12 +12 + 12 + 12

Real Life Application:

I need to buy eggs for the Boy Scout Jamboree. There will be 16 troops attending and I need a dozen eggs for each troop. How many eggs will I need?

Array Model of 16 X 12



16 X 12 can be broken apart in many ways to find the product:

10 X 16 = 160

2 X 16 = 32

12 X 10 = 120

12 X 6 = 72

12 x 10 = 120

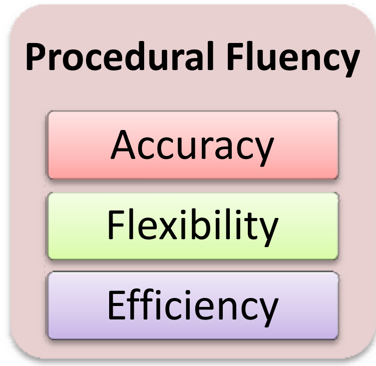
12 X 5 = 60

12 X 1 = 12

Stafford's goal of developing thoughtful, interested and involved mathematicians is grounded in the development of student conceptual understanding of mathematics. As Stafford students develop their conceptual understanding of mathematics, they take their first steps towards mathematical proficiency.

PROCEDURAL FLUENCY

Our mathematics curriculum is based on children solving problems with efficiency. A student knowing their math facts is an important component of both proficiency and our mathematics curriculum. Our curriculum supports students gaining a solid, practical understanding of mathematical concepts and how to fluently apply those understandings in new and changing situations. Procedural fluency is anchored in three core areas: (1) accuracy, (2) flexibility, and (3) efficiency.



Accuracy depends on several aspects of the problem-solving process, among them careful recording, knowledge of number facts and other important number relationships, and double-checking results. Research indicates that students using methods they understand make fewer errors than when strategies are learned without understanding.

Flexibility requires the knowledge of more than one approach to solving a particular kind of problem. Students need to be flexible in order to choose an appropriate strategy for the problem at hand, and also to use one method to solve a problem and another method to double-check the results.

Efficiency implies that the student does not get bogged down in too many steps or lose track of the logic of the strategy. An efficient strategy is one that the student can carry out easily, keeping track of sub-problems and making use of intermediate results to solve the problem.

There are many procedures a student might use to solve 16 X 12, these are but a few examples.

Use a known fact to derive the solution:

I know 12 x 12 is 144 and 4 x 12 is 48.

If I add these two products together I will have the solution for 16 X 12

Break the number apart:

16 X 10 = 160

16 X 2 = 32

Use a traditional procedure:

$$\begin{array}{r} {}^{1}16 \\ \underline{\times\ 12} \\ 32 \\ \underline{160} \\ \mathbf{192} \end{array}$$

STRATEGIC COMPETENCE

Strategic competence, historically known as problem solving, refers to the ability of a student to formulate mathematical problems, represent them, and solve them. When students develop a range of problem solving strategies they are not only deepening their understanding of the math, they are also increasing their ability to solve problems accurately. Taking time to develop a variety of strategies rather than relying on rote procedures results in a robust and long lasting understanding of mathematics. The end result also creates greater efficiency in mental computation and estimation.

Strategic competence begins in early childhood education with direct modeling strategies to solve mathematical problems, such as using manipulatives or a drawing to represent a problem. For example, a student might use tiles to model the addition problem $12 + 7$, or as they develop, draw arrows of different length and direction on a number line to model the addition of positive and negative integers, such as $^{-}4 + 5$.

As students gain mathematical competence, they will start to create student-invented strategies to solve problems. For example, consider the product of 64×8. A simple invented strategy might involve breaking the problem into two parts $60 \times 8 = 480$ and $4 \times 8 = 32$. This strategy demonstrates a powerful understanding of both number properties and procedure. As students continue to gain mathematical proficiency and develop fluency, they will be able to use and understand traditional algorithms. For example, students will be able to solve 386 x 215 using traditional algorithms *and* understand why the algorithm works.

As students become flexible thinkers in how they match the most efficient tool to the problem, they truly demonstrate their mathematical understanding and become mathematically proficient.

ADAPTIVE REASONING & PRODUCTIVE DISPOSITION

Adaptive reasoning refers to the ability to think logically about the relationships among concepts and situations. In mathematics, adaptive reasoning is the glue that holds everything together. One uses it to navigate through the many facts, procedures, concepts, and methods used to see that they all fit together in some way and make sense. Adaptive reasoning interacts with the other strands of proficiency, particularly during problem solving. Learners draw on their strategic competence to formulate and represent a problem.

Productive disposition refers to the power of students seeing the value of mathematics, perceiving it as both useful and worthwhile, believing that steady effort in learning mathematics pays off, and to seeing oneself as an effective learner and doer of mathematics. A productive disposition develops alongside each of the other strands. For example, as students build strategic competence in solving problems, their attitudes and beliefs about themselves as mathematics learners become more positive. The more mathematical concepts they understand, the more sensible mathematics becomes. When students see themselves as capable of learning mathematics and using it to solve problems, they further develop their procedural fluency and/or their adaptive reasoning abilities.

As students become more mathematically proficient in all five strands, their procedural fluency will also improve as a result, allowing students to be accurate, flexible and efficient mathematical thinkers.

LEARNING IS A CONTINUUM

Becoming mathematically proficient takes time and should be thought of as a long-term process that will be attained over a student's K-12 education. Stafford students will be well prepared for the math curriculum taught in middle school. Below is a brief overview of the K-5 math curriculum. The mathematical strands taught at all grade levels are:

- Number and Operations
- Geometry
- Patterns and Functions
- Data Analysis
- Measurement

Overview of K-5 math curriculum:

Kindergarten:

In Kindergarten, building a strong foundation in understanding number and geometric principles is crucial. Students use numbers to represent quantities and to solve quantitative problems, such as counting objects in a set and modeling simple joining and separating situations with objects. Students interpret the physical world with geometric ideas and describe it with corresponding vocabulary, such as circles, rectangles and triangles. Students in kindergarten work on ordering objects by measurable attributes such as length or weight in order to solve problems.

1st grade:

First grade students continue their work in building number sense and understanding about geometry. Students develop strategies for adding and subtracting whole numbers using a variety of models to form an understanding of the meanings of addition and subtraction. Students compare and order whole numbers up to 100 to develop an understanding of and solve problems involving relative sizes of these numbers. First grade students compose and decompose plane and solid figures, thus building an understanding of part-whole relationships.

2nd grade:

In second grade, students continue to build upon their foundation of number sense and begin to explore linear measurement. Students develop an understanding of the base-ten numeration system and place value concepts. Students develop a quick recall of basic addition facts and related subtraction facts. They solve arithmetic problems by applying their understanding of models of addition and subtraction. Students in second grade will develop an understanding of the meaning and processes of measurement, including underlying concepts like partitioning.

3rd Grade:

In third grade, students extend their understanding of numbers and operations to division, multiplication and fractions. Students will understand the meaning of multiplication and division of whole numbers through the use of representations. Students begin to develop an understanding of the meanings and uses of fractions to represent parts of a whole, parts of a set or points or distances on a number line. Third grade students will be able to describe and analyze two dimensional shapes by their sides and angles and connect these attributes to definitions of shapes.

4th Grade:

In fourth grade, students use understandings of multiplication to develop quick recall of the basic multiplication facts and related division facts. They apply their understanding of models for multiplication, place value, and properties of operations as they use efficient and generalized methods to multiply whole numbers. Students will develop an understanding of decimal notation as an extension of the base-ten system and connect this understanding to what they have learned about fractions. Students will develop an understanding of area and how to determine the areas of two dimensional shapes.

5th Grade:

In fifth grade, students develop an understanding of and fluency with addition and subtraction of fractions and decimals, using models to represent various problems. Students build on their understanding of multiplication by developing an understanding of and fluency with division of whole numbers. Students build on their previous experiences with two dimensional shapes by relating them to three dimensional shapes and analyzing their properties, including volume and surface area.

TRY THESE PROBLEMS

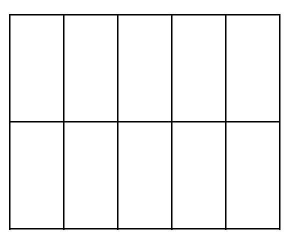
As you work through these problems, notice the different type of thinking involved.

Problem 1

What are the decimal and percent equivalents for the fractions $\frac{1}{2}$ and $\frac{2}{5}$?

Problem 2

Shade 4 small squares in a 2 x 5 rectangle. Using the rectangle, explain how to determine each of the following: (a) the percent of area that is shaded, (b) the decimal part of the area that is shaded, and (c) the fractional part of the area that is shaded.



FREQUENTLY ASKED QUESTIONS

I WANT TO SUPPORT MY CHILD WITH SCHOOL. WHERE DO I START?

- Provide a study place.
- Encourage your child to take risks and ask questions without concern for whether the answer is correct or not. Help your child focus on the parts of the problem they understand because they often know more than they think. Encouraging them in positive ways can make all the difference.
- When working with a story problem, encourage your child to focus on the situation and the action of the story. Often it is the action of the story that will give clues into which operation is needed to solve the problem.
- Encourage your child to carefully consider whether or not their answer is reasonable. Estimating a solution prior to solving will help students think about the reasonableness of their answer.
- Help with organizing and maintaining a notebook.
- Help your child develop a system for writing down assignments, as well as keeping track of progress.
- Encourage your child to identify study buddies or another math student she/he can call to work with on assignments, get clarification, and to find out about makeup work when needed.
- Encourage and expect your child to get work done on time, to stay caught up, to get help in a timely manner, and to correct errors in work.

HOW CAN I BECOME MORE FAMILIAR WITH THE MATH BEING TAUGHT IN CLASS?

Curriculum night is a great place to start with understanding the math your child will engage with over each school year. Teachers use this evening to talk about their instructional focus, the concepts they will cover over the year and how you can support your child. Other avenues to become familiar with your child's experience are through the homework your child brings home, conferences and engaging your child in conversation about their mathematical thinking. There are also student handbooks for 1st grade through 5th grade that give an overview of the math at a particular grade level. These handbooks show different strategies and explain a variety of games and activities that you can do with your child that are similar to those they will experience in class.

WHEN IS IT APPROPRIATE FOR MY CHILD TO USE A CALCULATOR?

If the primary purpose of the instructional activity is to practice computational skills, students should not be using a calculator. For example, if the instructional activity is practicing adding two multi-digit numbers together, students should not use a calculator.

Students should be allowed to use calculators when the goal of the instructional activity is not to compute, but computation is involved in the problem solving. For example, if the instructional activity is to examine the pattern created by multiplying various whole numbers together, a calculator may be used since the task is about exploring the pattern and not the computation of the numbers.

In general, students should have full access to calculators when they are exploring patterns, conducting investigations, testing conjectures, and solving problems.

Helping students know when to use a calculator and when not to use one is precisely the work of the teacher. Don't hesitate to contact your child's teacher if you have a question about your child's calculator use for school work.

WHAT ARE BEST PRACTICES IN TEACHING MATH?

"Best Practices in Teaching Mathematics" encompasses everything your child's teacher does within the classroom.

An effective math classroom is filled with students engaged in important mathematics and lessons are designed to enhances student's understanding and develop students' capacity to do math successfully. In order to build a classroom where students are confidently engaged in complex mathematical concepts, four practices are evident:

- All Students are engaged in worthwhile activities that focus on mathematical understanding, inventions and sense making.
- The classroom's culture provides opportunities for inquiry, wrong answers, collaboration and disequilibrium, leading to mathematical learning by all students.
- The tasks in which students are engaged are mathematically worthwhile for all students.
- The teacher's knowledge of the mathematics content they are teaching and the trajectory of that content enables the teacher to support lasting student understanding.

It is developing a strong mathematical understanding that lies at the center of our work with children. Stafford's teachers are doing great work with each other and with their students to continue to foster the environments and practices that press and challenge each student in their classrooms.

WHAT QUESTIONS SHOULD I ASK MY CHILD TO SUPPORT THEM?

To help students build confidence and rely on their own understanding, ask…

- Why is that true?
- How did you reach that conclusion?
- Does that make sense?
- Can you make a model to show that?
- Can you solve the problem using a different strategy?

To promote problem solving, ask…

- What do you need to find out?
- What information do you have?
- What strategies are you going to use?
- What tools or models will you need?
- What do you think the answer or result will be?

  Will a calculator help with the understanding of the problem?

To help when students get stuck, ask…

- How would you describe the problem in your own words?
- What part of the problem do you understand?
- How did you tackle similar problems?
- Could you try it with simpler numbers?
- Would it help to create a diagram? Make a table?
- Can you guess and check?
- Would it help to draw a picture? Make a model?

WHAT ACTIVITIES CAN I DO WITH MY CHILD TO SUPPORT LEARNING MATH?

The best way to help support your child learning math at home is through the use of games and real-life applications. Below are some examples.

Real-Life Applications:

- Counting out money at the grocery store
- Scaling up or down a recipe when baking (i.e. the recipe calls for 2 eggs; how many eggs do we need since we are tripling the batch of cookies?)
- Web-page design
- Sorting various objects into groups
- Activities such as Legos or building a birdhouse

Games:

- High/Low – Guess the number
- Monopoly and Monopoly Jr.
- Chutes & Ladders
- Sudoku
- Checkers
- Fraction Tracks
- Yahtzee
- Blokus

Online games:

- Grades K & 1: http://investigations.terc.edu/library/Games_K1.cfm
- Grades 2 & 3: http://investigations.terc.edu/library/Games_23.cfm
- Grades 4 & 5: http://investigations.terc.edu/library/Games_45.cfm

HOW CAN I LEARN ABOUT MY CHILD'S PROGRESS IN MATH?

If you have a question about your child's progress in math, you should contact their teacher directly. Here are some questions you can ask their teacher to get you started:

- Can I schedule a conference with you about my child's progress? How?
- Will my child take any state or national tests this year?
- What can I do to help you?
- How is progress in learning math measured?

WHO SHOULD I TALK TO IF I HAVE MORE QUESTIONS?

Do you still have questions about Stafford's math program?

We recommend you contact:

- Your child's teacher
- Patrick Minor, instructional coordinator
- Nancy Curtis, math specialist
- Patrick Meigs, principal

SOLUTIONS AND DISCUSSION TO "TRY THESE PROBLEMS"

Problem 1

What are the decimal and percent equivalents for the fractions $\frac{1}{2}$ and $\frac{2}{5}$?

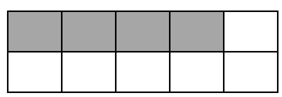
- The decimal equivalent of $\frac{1}{2}$ is 0.5. This can be written as 50%.
- The decimal equivalent of $\frac{2}{5}$ is 0.4. This can be written as 40%.

Discussion

This problem requires students to recall the decimal and percent equivalents as simple facts. Students are not expected to explain or demonstrate how they derived their answers. Students may have used a strategy to find the answer, but they did not have to explain or show their process.

Problem 2

Shade 4 small squares in a 2 x 5 rectangle. Using the rectangle, explain how to determine each of the following: (a) the percent of area that is shaded, (b) the decimal part of the area that is shaded, and (c) the fractional part of the area that is shaded.



Possible answers

a) The 2 x 5 rectangle has a total of 10 squares. Each of the squares must be 10% since the whole rectangle is 100%. If 4 of the squares are shaded, then the shaded part must be 40%.
b) If the whole rectangle is equal to 1 and is made of 10 squares, I would need to divide the rectangle by 10 to find how much each square is. Each of the squares is equal to 0.1 of the rectangle. Since there are 4 shaded squares, I would multiply 0.1 x 4 which equals 0.4.
c) There are 4 shaded squares out of 10 squares total. Since a fraction compares one thing to another, this can be written as 4 of 10, or $\frac{4}{10}$ as a fraction.

Discussion

In this problem, students are asked to use the rectangle as a visual model of the problem. They are then asked to explain the connections between the visual model and the problems. This task requires students to do more than just recall facts, it requires students to demonstrate an understanding of the concepts.